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ON THE NUMBERS 11 … 122 … 25 AND 44…488…89

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ABSTRACT: In this paper, two numbers 11…122…25 and 44…488…89 where 1 and 8 are repeated n times and 2 and 4 are repeated $n+1$ times have been discussed. It has been shown that these two numbers are perfect square.

INTRODUCTION

B. Sury (2016) proposed a problem that the number 11…122…25 where 1 is repeated 2015 times and 2 is repeated 2016 times is a perfect square. **Hari Kishan** (2016) communicated a solution of this problem.
Here it has been shown that the above two numbers are perfect square for all positive integral values of n.

ANALYSIS

The given numbers are discussed in the following two cases:

Case 1: The given number 11…122…25 contains n digit 1 and $n+1$ digit 2. Thus the given number can be written as

$$11 \ldots 122 \ldots 25 = 5 + 2 \times 10 + 2 \times 10^2 + \cdots 2 \times 10^{n+1} + 10^{n+2} + \cdots 10^{2n+1}$$
$$= 5 + \frac{2\times10(10^{n+1}-1)}{10-1} + \frac{10^{n+2}(10^n-1)}{10-1}$$
$$= \frac{45+2\times10^{n+2}-20+10^{2n+2}-10^{n+2}}{9}$$
$$= \frac{25+10^{n+2}+10^{2n+2}}{9}$$
$$= \left(\frac{10^{n+1}+5}{3}\right)^2.$$

This shows that the given number is a perfect square.
The expression in the bracket $\frac{10^{n+1}+5}{3}$ can be shown an integer by mathematical induction. For $n=1$, the expression becomes $\frac{100+5}{3} = 35$ which is an integer.
Suppose the expression is an integer for $n=m$ i.e. $\frac{10^{m+1}+5}{3} = p$ (say) is an integer. This gives $10^{m+1} = 3p-5$.
Now for $n = m+1$, we have

$\frac{10^{m+2}+5}{3} = \frac{10\times10^{m+1}+5}{3} = \frac{10(3p-5)+5}{3} = \frac{30p-45}{3} = 10p-15$ is an integer. Hence by mathematical induction, the expression $\frac{10^{n+1}+5}{3}$ is an integer.

Case 2: The given number 44…488…89 contains $n+1$ digit 4 and n digit 8. Thus the given number can be written as

$$44 \ldots 488 \ldots 89 = 9 + 8 \times 10 + 8 \times 10^2 + \cdots 8 \times 10^n + 4 \times 10^{n+1} + \cdots 4 \times 10^{2n+2}$$

$$= 9 + \frac{8\times10(10^{n}-1)}{10-1} + \frac{4\times10^{n+1}(10^{n+1}-1)}{10-1}$$

$$= \frac{81+8\times10^{n+1}-80+4\times10^{2n+2}-4\times10^{n+1}}{9}$$

$$= \frac{1+4\times10^{n+1}+4\times10^{2n+2}}{9}$$

$$= \left(\frac{2\times10^{n+1}+1}{3}\right)^{2}.$$

This shows that the given number is a perfect square. As in Case 1, it can be shown that the expression in the bracket is an integer.

REFERENCES

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